Using an electrocardiogram to measure the electric dipole moment of the heart

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Abstract

Using a relatively simple electrocardiogram (EKG) made of two operational amplifiers working simultaneously, we were able to measure the max electric dipole moment of the heart of a human using three trials, which was found to be $1.68 \pm 0.01 \times 10^{-15}$ Cm. This max electric dipole moment was converted into an equivalent resultant dipole moment of $1.186 \pm 0.009 \times 10^{-5}$ Am. This value was compared to that found by Nelson et al. which was 2.32×10^{-5} Am and the discrepancy was found to not be significant.

Introduction

An electrocardiogram (EKG) measures the voltage differences caused by the electric dipole moment (EDM) $\vec{p}$ of the heart. An EDM describes a symmetrical distribution of electric charges. If there is a distribution of positive charge *q* and negative charge -*q* separated by a distance *d*, the EDM points from the negative charges to the positive charges. The equation for this EDM is given by

$$\vec{p} = q\vec{d}$$

The voltage due to the EDM at a distance *r* from it is given by

$$V = \frac{1}{4\pi\epsilon_0\kappa}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

Nelson et al. in their data found what they called the resultant dipole moment (RDM) $\overrightarrow{m}$ and used this to find the voltage. The voltage due to the RDM at a distance *r* is given by

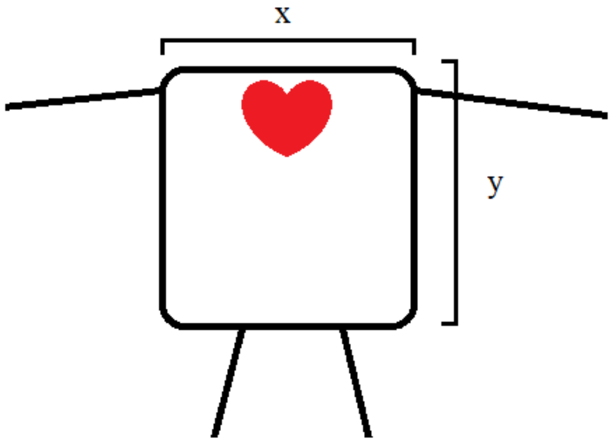
$$V = \frac{1}{4\pi\sigma}\frac{\overrightarrow{m}\cdot\vec{r}}{r^3}$$

The EDM and the RDM are related by

$$\frac{\sigma}{\epsilon_0\kappa}\vec{p} = \overrightarrow{m}$$

Methods: Set-up

Several assumptions were made in our model. The body of the subject was considered to be an empty cavity of water, which has a κ value of 80. The heart was placed directly in the middle of the cavity in line with the shoulders. The arms and legs were considered perfect conductors so there would be no voltage drop across them. The right leg was set to ground. The positions of the arms were $\vec{r} = \pm\frac{x}{2}\hat{x}$ and the positions of the legs were both at $\vec{r} = -y\hat{y}$



Methods: x-component of EDM

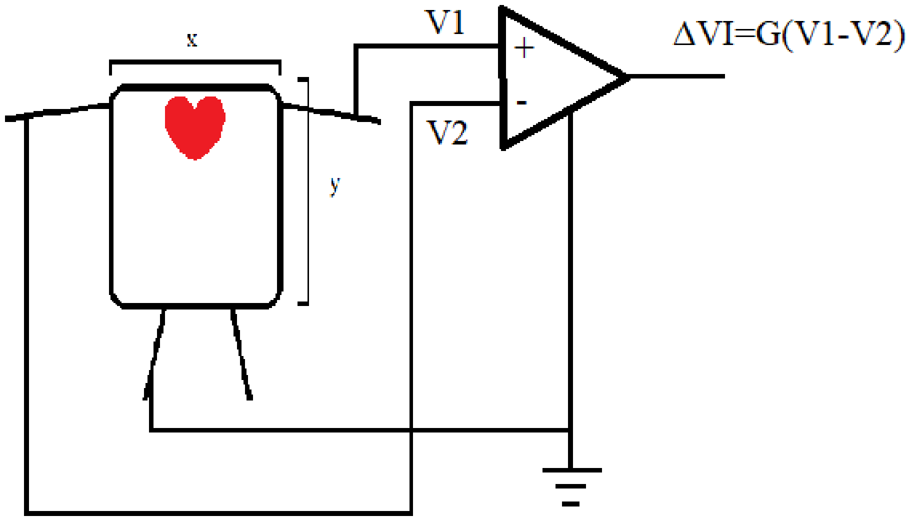
An operational amplifier (op amp) was used to find the x-component of EDM. The output of the op amp ΔVI was the difference between the voltages found at the arms multiplied by some gain G, which in the experiment was set to 100.

$$V1 = \frac{1}{4\pi\epsilon_0\kappa}\frac{\vec{p}\cdot\vec{r}}{r^3} = \frac{1}{4\pi\epsilon_0\kappa}\frac{p_x\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)^3} = \frac{1}{\pi\epsilon_0\kappa}\frac{p_x}{x^2}$$

$$V2 = -\frac{1}{4\pi\epsilon_0\kappa}\frac{p_x\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)^3} = -\frac{1}{\pi\epsilon_0\kappa}\frac{p_x}{x^2}$$

$$\Delta VI = G(V1 - V2) = G\left(\frac{2}{\pi\epsilon_0\kappa}\frac{p_x}{x^2}\right)$$

$$p_x = \frac{\pi\epsilon_0\kappa x^2}{2G}\Delta VI$$



Methods: y-component of EDM

Using an op amp with the same gain of 100, the voltage at the right arm and the left leg was found

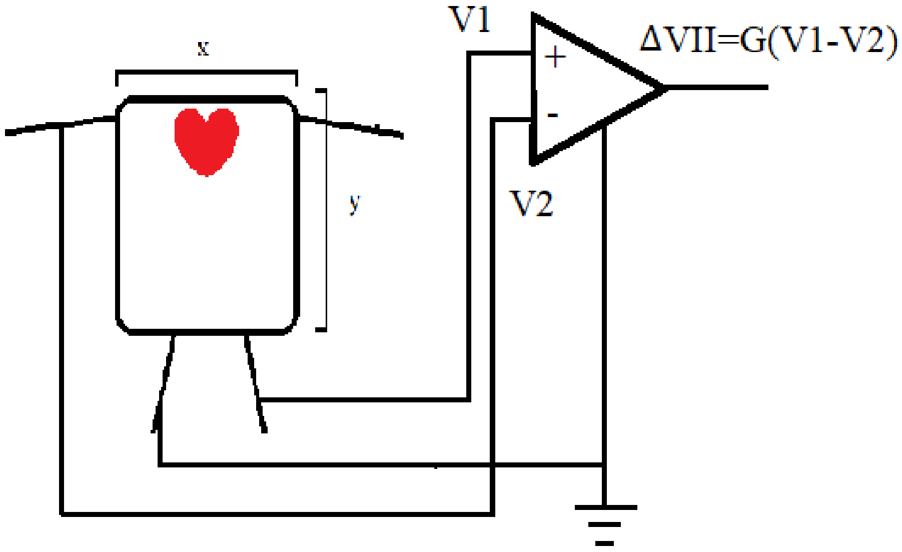
$$V1 = -\frac{1}{4\pi\epsilon_0\kappa}\frac{p_y y}{y^3} = -\frac{1}{4\pi\epsilon_0\kappa}\frac{p_y}{y^2}$$

$$V2 = -\frac{1}{4\pi\epsilon_0\kappa}\frac{p_x\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)^3} = -\frac{1}{\pi\epsilon_0\kappa}\frac{p_x}{x^2}$$

$$\Delta VII = G(V1 - V2)$$

$$\Delta VII = G\left(-\frac{1}{4\pi\epsilon_0\kappa}\frac{p_y}{y^2} + \frac{1}{\pi\epsilon_0\kappa}\frac{p_x}{x^2}\right)$$

$$p_y = -\frac{4\pi\epsilon_0\kappa y^2}{G}\left(\Delta VII - \frac{\Delta VI}{2}\right)$$



Methods

Two op amps were used to measure ΔVI and ΔVII simultaneously over ten seconds. 5000 samples of the voltage were sampled over that time. 3 trials were conducted, twice for the same male person and once with a female person.

From the x- and y-component of the EDM, the total magnitude of the EDM could be found by

$$p = \sqrt{p_x^2 + p_y^2}$$

Results

	TRIAL 1		TRIAL 2		TRIAL 3	
MAXIMA	p (Cm)	m (Am)	p (Cm)	m (Am)	p (Cm)	m (Am)
1	2.03009E-15	1.43339E-05	4.37288E-15	3.08757E-05	1.429E-15	1.00898E-05
2	1.91959E-15	1.35537E-05	2.65721E-15	1.87619E-05	1.33115E-15	9.39891E-06
3	1.88715E-15	1.33246E-05	2.38389E-15	1.6832E-05	1.10917E-15	7.83158E-06
4	1.88365E-15	1.33E-05	2.35204E-15	1.66071E-05	1.08985E-15	7.69515E-06
5	1.87156E-15	1.32146E-05	2.30397E-15	1.62677E-05	1.07824E-15	7.61313E-06
6	1.83849E-15	1.29811E-05	2.24029E-15	1.5818E-05	1.07773E-15	7.60958E-06
7	1.82048E-15	1.28539E-05	2.11116E-15	1.49063E-05	1.07598E-15	7.59723E-06
8	1.80605E-15	1.2752E-05	2.10891E-15	1.48905E-05	1.0469E-15	7.39187E-06
9	1.78983E-15	1.26375E-05	2.10891E-15	1.48905E-05	1.01563E-15	7.17112E-06
10	1.77361E-15	1.2523E-05	2.07815E-15	1.46733E-05	9.66987E-16	6.82763E-06
11	1.75801E-15	1.24128E-05	2.07704E-15	1.46654E-05	9.50773E-16	6.71315E-06
12	1.75455E-15	1.23884E-05	2.06194E-15	1.45588E-05	9.47406E-16	6.68938E-06
13	1.74242E-15	1.23028E-05	2.04873E-15	1.44655E-05	9.20209E-16	6.49734E-06
14	1.69375E-15	1.19591E-05	2.04573E-15	1.44444E-05	9.03983E-16	6.38278E-06
15	1.69193E-15	1.19463E-05	2.04517E-15	1.44404E-05	8.88433E-16	6.27298E-06
16	1.6795E-15	1.18585E-05	2.03067E-15	1.4338E-05	8.71533E-16	6.15366E-06
17	1.67883E-15	1.18537E-05	2.01566E-15	1.4232E-05	8.67744E-16	6.1269E-06
18	1.67753E-15	1.18446E-05	2.01331E-15	1.42154E-05	8.57408E-16	6.05393E-06
19	1.6626E-15	1.17392E-05	2.01069E-15	1.41969E-05	8.49283E-16	5.99656E-06
20	1.66009E-15	1.17215E-05	1.99944E-15	1.41175E-05	8.48981E-16	5.99443E-06

Twenty maxima for the EDM were found and their average was taken

Trial	Avg max p (Cm)	Error	Avg max m (Am)	Error
1	1.781E-15	5E-18	1.258E-05	3.6E-08
2	2.3E-15	3E-17	1.59E-05	2E-07
3	1.006E-15	8E-18	7.105E-06	6E-08
All	1.68E-15	1E-17	1.186E-05	9E-08

Average and error for three trials separately and together

Conclusion

Nelson et al. have a table that compares the body weight of animals to their RDM. From linear regression, the equation for the relation between body weight and *m* is

$$\log(m) = -5.93 + 0.797\log(wt)$$

with an error in each log(m) of ± 0.2.

Nelson et al.'s log(m) for man is -4.6 ± 0.2

The log(m) of our value for *m* is

-4.926 ± 0.008

The significance ratio is

$$\frac{|-4.6 + 4.926|}{0.2 + 0.008} = 1.567 < 3$$

Species	Number	Heart wt (g)	Body wt (kg)	m (Am)	ln(m)
Leopard frog	4	0.16	0.036	5.00E-08	-7.30
Hamster	2	0.44	0.1	1.50E-07	-6.82
Turtle	19	0.96	0.352	5.10E-07	-6.29
Rat	2	1.1	0.277	1.07E-06	-5.97
Bullfrog	7	1.25	0.39	4.90E-07	-6.31
Rabbit	12	10.6	3.5	3.55E-06	-5.45
Monkey	12	42.8	8.65	4.78E-06	-5.32
Pig	51	51	12	6.65E-06	-5.18
Dog	17	108	14.2	1.63E-05	-4.79
Lamb	1	122	25	2.13E-05	-4.67
Man	15	300	71.5	2.32E-05	-4.63
Horse	4	3060	419	1.30E-04	-3.89

References

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Acknowledgments

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